ECE 312
Electronic Circuits (A)

Lec. 6: BJT Modeling and re Transistor Model (small signal analysis) (2)

Instructor

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Agenda

The r_e Transistor Model (small signal analysis) (2)

Emitter Follower (Common Collector) Configuration

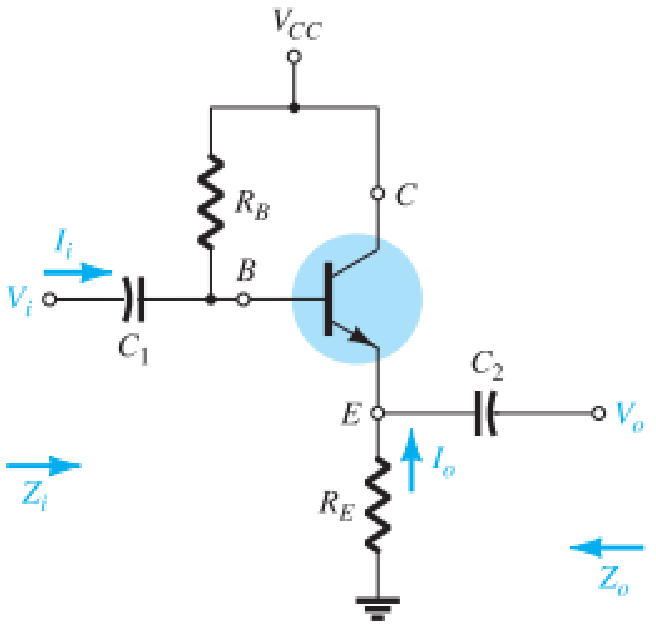


FIG. 5.36
Emitter-follower configuration.

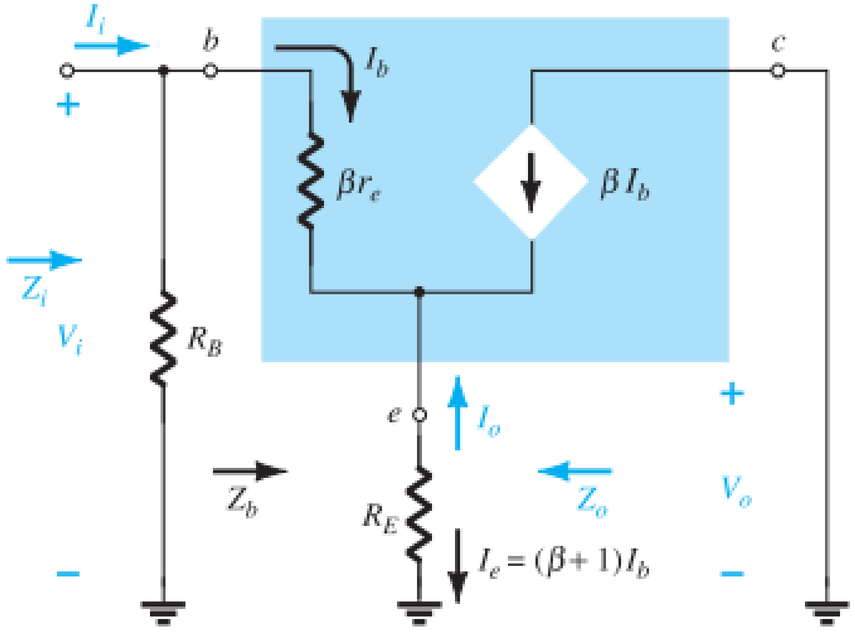


FIG. 5.37
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.36.

$$Z_i = R_B \| Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$$Z_b \cong \beta R_E$$

$R_E \gg r_e$

Emitter Follower Configuration (o/p impedance and Gain)

$$I_e = (\beta + 1)I_b = (\beta + 1)\frac{V_i}{Z_b}$$

$$I_e = \frac{(\beta + 1)V_i}{\beta r_e + (\beta + 1)R_E}$$

$$I_e = \frac{V_i}{[\beta r_e/(\beta + 1)] + R_E}$$

$$(\beta + 1) \cong \beta$$

$$\frac{\beta r_e}{\beta + 1} \cong \frac{\beta r_e}{\beta} = r_e$$

$$I_e \cong \frac{V_i}{r_e + R_E}$$

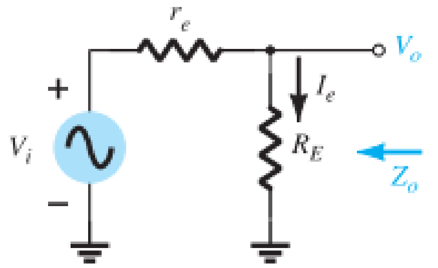


FIG. 5.38
Defining the output impedance for the emitter-follower configuration.

$$Z_o = R_E \| r_e$$

$$Z_o \cong r_e$$

$$V_o = \frac{R_E V_i}{R_E + r_e}$$

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

Because R_E is usually much greater than r_e,

$$R_E + r_e \cong R_E;$$

$$A_v = \frac{V_o}{V_i} \cong 1$$ in phase

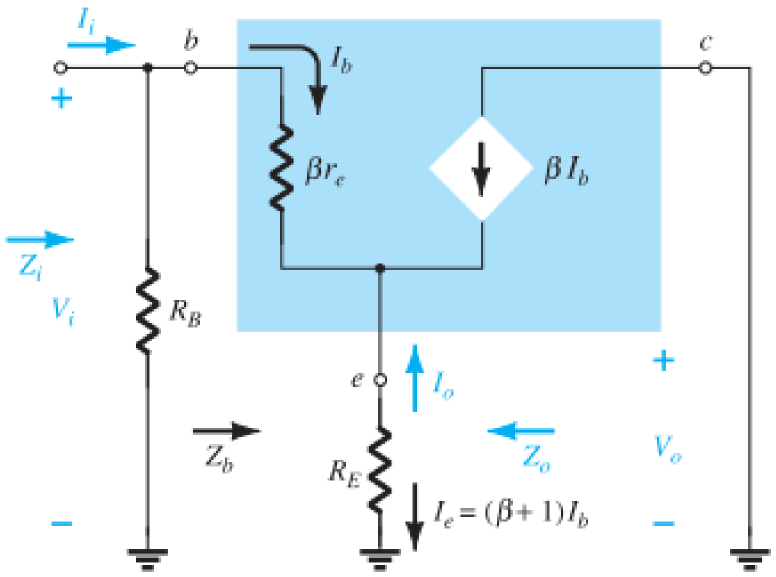


FIG. 5.37
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.36.

Emitter Follower Configuration..

Effect of r_o

$$Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \dfrac{R_E}{r_o}}$$

$$r_o \geq 10R_E$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$r_o \geq 10R_E$

$$Z_o = r_o \| R_E \| \frac{\beta r_e}{(\beta + 1)}$$

$$Z_o = r_o \| R_E \| r_e$$

$$Z_o \cong R_E \| r_e$$

Any r_o

$$Z_o \cong r_e$$

$$A_v = \frac{(\beta + 1)R_E/Z_b}{1 + \dfrac{R_E}{r_o}}$$

$$A_v \cong \frac{\beta R_E}{Z_b}$$

$$Z_b \cong \beta(r_e + R_E)$$

$$A_v \cong \frac{\beta R_E}{\beta(r_e + R_E)}$$

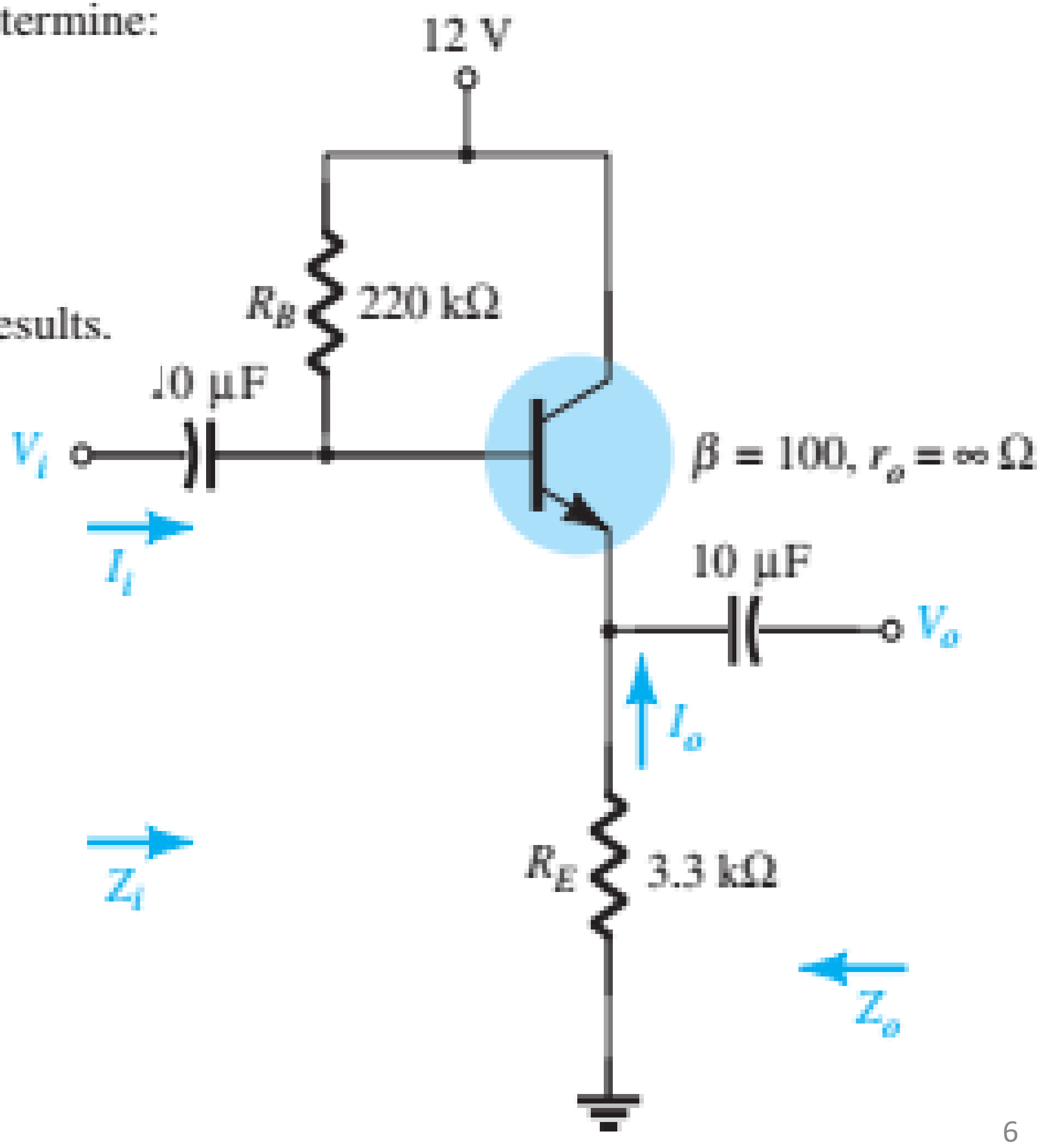
$$A_v \cong \frac{R_E}{r_e + R_E}$$

$r_o \geq 10R_E$

Emitter Follower Configuration (Example)

EXAMPLE 5.7 For the emitter-follower network of Fig. 5.39, determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.
e. Repeat parts (b) through (d) with $r_o = 25\ \text{k}\Omega$ and compare results.



Emitter Follower Configuration (Example (wo/ro)

EXAMPLE 5.7 For the emitter-follower network of Fig. 5.39, determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.
e. Repeat parts (b) through (d) with $r_o = 25\ \text{k}\Omega$ and compare results.

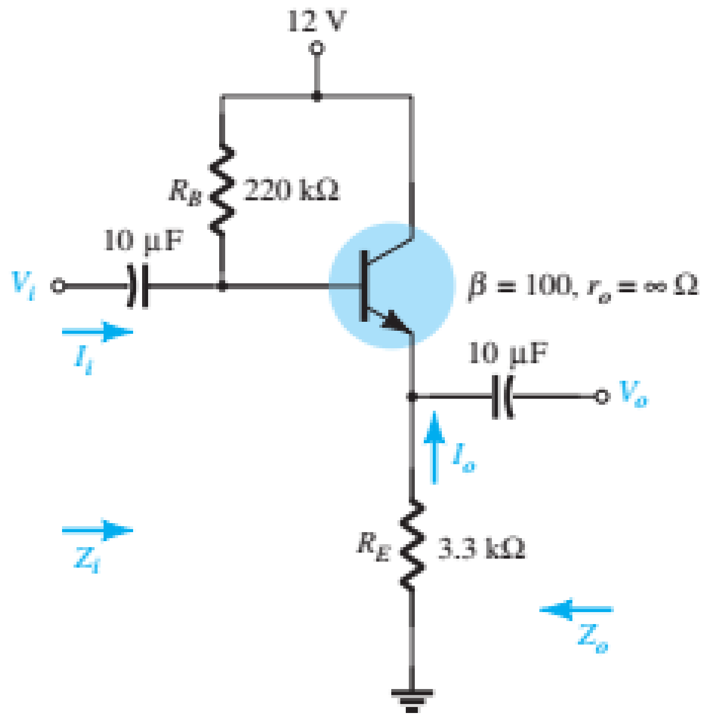


FIG. 5.39

Solution:

a.
$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{12\ \text{V} - 0.7\ \text{V}}{220\ \text{k}\Omega + (101)3.3\ \text{k}\Omega} = 20.42\ \mu\text{A}$$
$$I_E = (\beta + 1)I_B = (101)(20.42\ \mu\text{A}) = 2.062\ \text{mA}$$
$$r_e = \frac{26\ \text{mV}}{I_E} = \frac{26\ \text{mV}}{2.062\ \text{mA}} = \mathbf{12.61\ \Omega}$$

b.
$$Z_b = \beta r_e + (\beta + 1)R_E = (100)(12.61\ \Omega) + (101)(3.3\ \text{k}\Omega) = 1.261\ \text{k}\Omega + 333.3\ \text{k}\Omega = 334.56\ \text{k}\Omega \cong \beta R_E$$
$$Z_i = R_B \| Z_b = 220\ \text{k}\Omega \| 334.56\ \text{k}\Omega = \mathbf{132.72\ k\Omega}$$

c.
$$Z_o = R_E \| r_e = 3.3\ \text{k}\Omega \| 12.61\ \Omega = \mathbf{12.56\ \Omega} \cong r_e$$

d.
$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e} = \frac{3.3\ \text{k}\Omega}{3.3\ \text{k}\Omega + 12.61\ \Omega} = \mathbf{0.996 \cong 1}$$

Emitter Follower Configuration (Example w/ro)

e. Checking the condition $r_o \geq 10R_E$, we have

$$25\ \text{k}\Omega \geq 10(3.3\ \text{k}\Omega) = 33\ \text{k}\Omega$$

which is *not* satisfied. Therefore,

$$Z_b = \beta r_e + \frac{(\beta + 1)R_E}{1 + \dfrac{R_E}{r_o}} = (100)(12.61\ \Omega) + \frac{(100 + 1)3.3\ \text{k}\Omega}{1 + \dfrac{3.3\ \text{k}\Omega}{25\ \text{k}\Omega}}$$

$$= 1.261\ \text{k}\Omega + 294.43\ \text{k}\Omega$$

$$= 295.7\ \text{k}\Omega$$

with $Z_i = R_B \| Z_b = 220\ \text{k}\Omega \| 295.7\ \text{k}\Omega$

$$= \mathbf{126.15\ k\Omega} \text{ vs. } 132.72\ \text{k}\Omega \quad \text{obtained earlier}$$

$$Z_o = R_E \| r_e = \mathbf{12.56\ \Omega} \quad \text{as obtained earlier}$$

$$A_v = \frac{(\beta + 1)R_E/Z_b}{\left[1 + \dfrac{R_E}{r_o}\right]} = \frac{(100 + 1)(3.3\ \text{k}\Omega)/295.7\ \text{k}\Omega}{\left[1 + \dfrac{3.3\ \text{k}\Omega}{25\ \text{k}\Omega}\right]}$$

$$= \mathbf{0.996 \cong 1}$$

matching the earlier result.

Emitter Follower Configuration (Diff. Biasing)

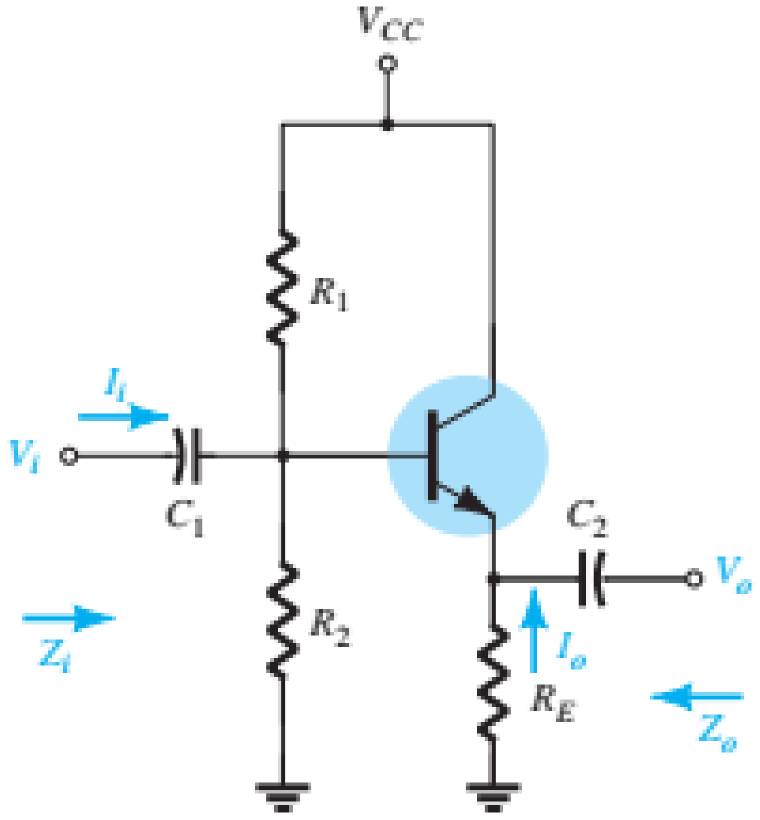


FIG. 5.40
Emitter-follower configuration with a voltage-divider biasing arrangement.

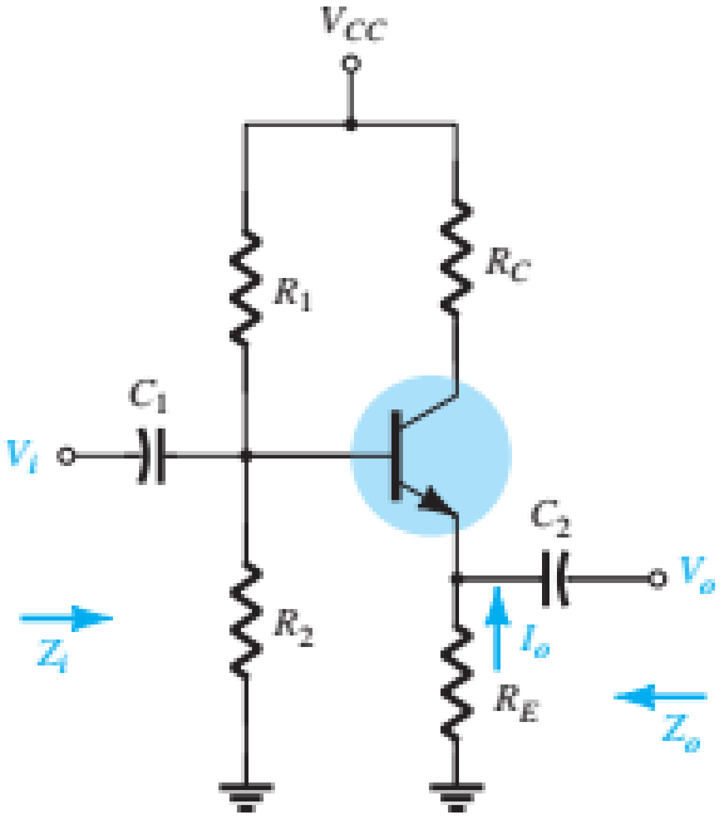


FIG. 5.41
Emitter-follower configuration with a collector resistor R_C.

Common-Base Configuration

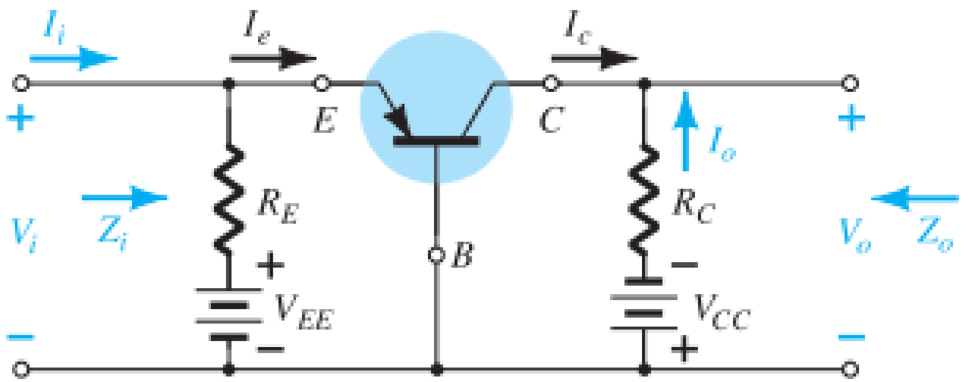


FIG. 5.42
Common-base configuration.

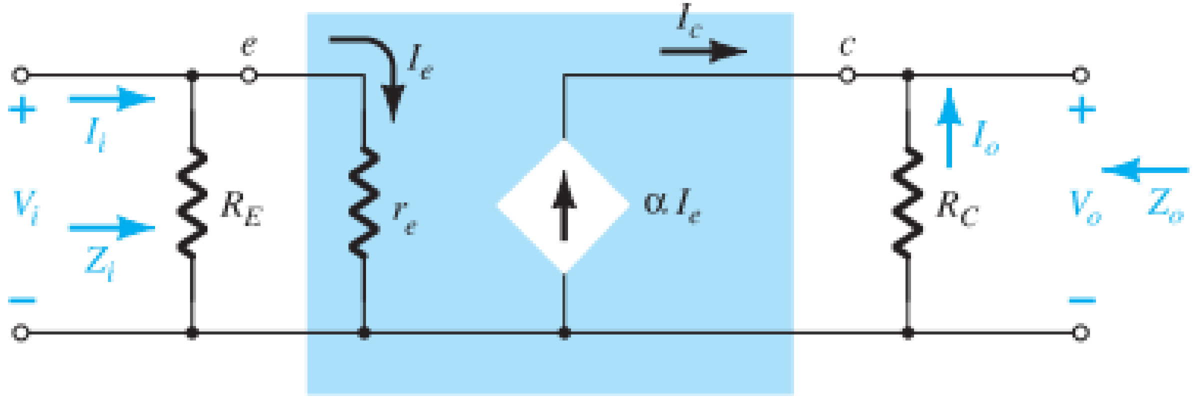


FIG. 5.43
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.44.

$$Z_i = R_E \| r_e$$

$$Z_o = R_C$$

$$I_e = I_i$$

$$I_o = -\alpha I_e = -\alpha I_i$$

$$A_i = \frac{I_o}{I_i} = -\alpha \cong -1$$

$$V_o = -I_o R_C = -(-I_c)R_C = \alpha I_e R_C$$

$$I_e = \frac{V_i}{r_e}$$

$$V_o = \alpha\left(\frac{V_i}{r_e}\right)R_C$$

$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \cong \frac{R_C}{r_e}$$

Phase Relationship The fact that A_v is a positive number shows that V_o and V_i are in phase for the common-base configuration.

Effect of r_o For the common-base configuration, $r_o = 1/h_{ob}$ is typically in the megohm range and sufficiently larger than the parallel resistance R_C to permit the approximation $r_o \| R_C \cong R_C$.

Common-Base Configuration (Example)

EXAMPLE 5.8 For the network of Fig. 5.44, determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.
e. A_i.

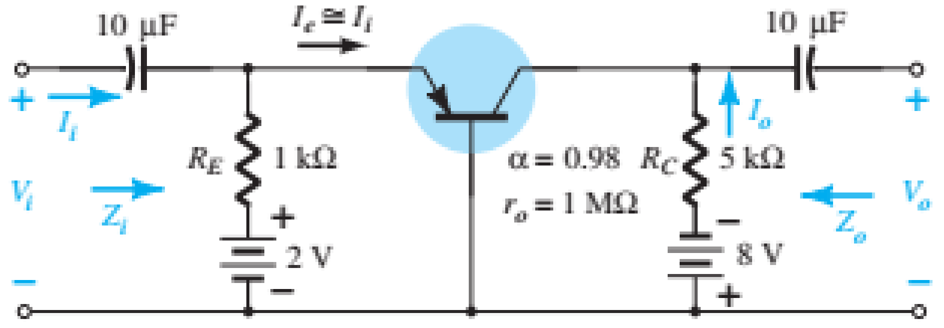


FIG. 5.44

Solution:

a. $I_E = \dfrac{V_{EE} - V_{BE}}{R_E} = \dfrac{2\text{ V} - 0.7\text{ V}}{1\text{ k}\Omega} = \dfrac{1.3\text{ V}}{1\text{ k}\Omega} = 1.3\text{ mA}$

$r_e = \dfrac{26\text{ mV}}{I_E} = \dfrac{26\text{ mV}}{1.3\text{ mA}} = \mathbf{20\ \Omega}$

b. $Z_i = R_E \| r_e = 1\text{ k}\Omega \| 20\ \Omega$

$= \mathbf{19.61\ \Omega} \cong r_e$

c. $Z_o = R_C = \mathbf{5\text{ k}\Omega}$

d. $A_v \cong \dfrac{R_C}{r_e} = \dfrac{5\text{ k}\Omega}{20\ \Omega} = \mathbf{250}$

e. $A_i = \mathbf{-0.98} \cong -1$

Collector-Feedback Configuration

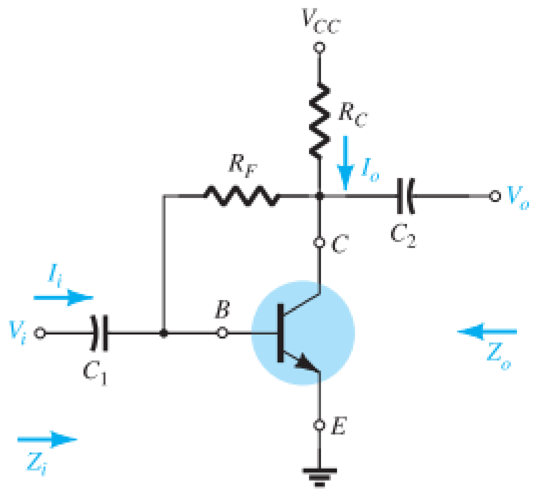


FIG. 5.45
Collector feedback configuration.

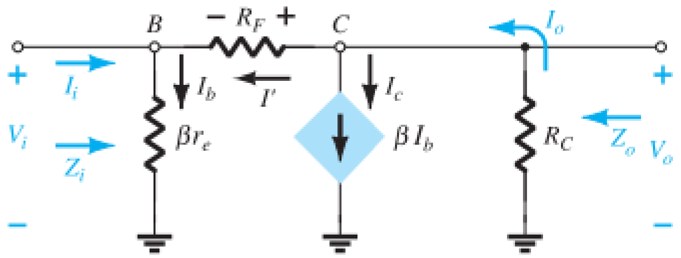


FIG. 5.46
Substituting the r_e equivalent circuit into the ac equivalent network of F

$$I_o = I' + \beta I_b$$

$$I' = \frac{V_o - V_i}{R_F}$$

$$V_o = -I_o R_C = -(I' + \beta I_b)R_C$$

$$V_i = I_b \beta r_e$$

$$I' = -\frac{(I' + \beta I_b)R_C - I_b \beta r_e}{R_F} = -\frac{I' R_C}{R_F} - \frac{\beta I_b R_C}{R_F} - \frac{I_b \beta r_e}{R_F}$$

$$I'\left(1 + \frac{R_C}{R_F}\right) = -\beta I_b \frac{(R_C + r_e)}{R_F}$$

$$I' = -\beta I_b \frac{(R_C + r_e)}{R_C + R_F}$$

$$I_i = I_b - I' = I_b + \beta I_b \frac{(R_C + r_e)}{R_C + R_F}$$

$$I_i = I_b\left(1 + \beta \frac{(R_C + r_e)}{R_C + R_F}\right)$$

$$Z_i = \frac{V_i}{I_i} = \frac{I_b \beta r_e}{I_b\left(1 + \beta \frac{(R_C + r_e)}{R_C + R_F}\right)} = \frac{\beta r_e}{1 + \beta \frac{(R_C + r_e)}{R_C + R_F}}$$

$$R_C \gg r_e \qquad Z_i = \frac{\beta r_e}{1 + \frac{\beta R_C}{R_C + R_F}}$$

$$Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_C + R_F}}$$

Collector-Feedback Configuration

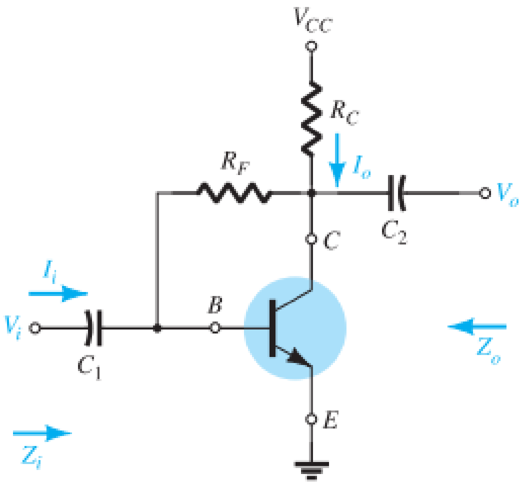


FIG. 5.45
Collector feedback configuration.

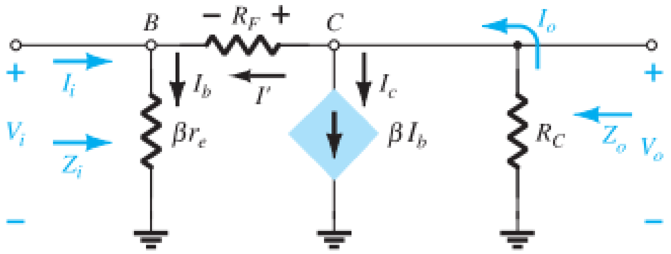


FIG. 5.46
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.45.

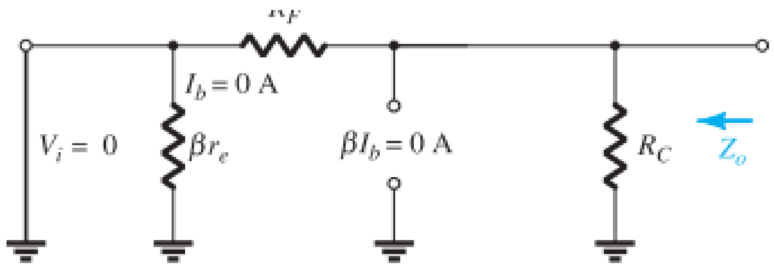


FIG. 5.47
Defining Z_o for the collector feedback configuration.

$$Z_o \cong R_C \| R_F$$

$$V_o = -I_o R_C = -(I' + \beta I_b)R_C$$

$$= -\left(-\beta I_b \frac{(R_C + r_e)}{R_C + R_F} + \beta I_b\right)R_C$$

$$V_o = -\beta I_b\left(1 - \frac{(R_C + r_e)}{R_C + R_F}\right)R_C$$

$$A_v = \frac{V_o}{V_i} = \frac{-\beta \cancel{I_b}\left(1 - \frac{(R_C + r_e)}{R_C + R_F}\right)R_C}{\beta r_e \cancel{I_b}}$$

$$= -\left(1 - \frac{(R_C + r_e)}{R_C + R_F}\right)\frac{R_C}{r_e}$$

$$A_v = -\left(1 - \frac{R_C}{R_C + R_F}\right)\frac{R_C}{r_e}$$

$$A_v = -\frac{(\cancel{R_C} + R_F - \cancel{R_C})}{R_C + R_F}\frac{R_C}{r_e}$$

$$A_v = -\left(\frac{R_F}{R_C + R_F}\right)\frac{R_C}{r_e}$$

$$A_v \cong -\frac{R_C}{r_e}$$ 180° phase shift

Collector-Feedback Configuration..

Effect of r_o

$$Z_i = \frac{1 + \dfrac{R_C \| r_o}{R_F}}{\dfrac{1}{\beta r_e} + \dfrac{1}{R_F} + \dfrac{R_C \| r_o}{\beta r_e R_F} + \dfrac{R_C \| r_o}{R_F r_e}}$$

$r_o \geq 10R_C$:

$$Z_i = \frac{1 + \dfrac{R_C}{R_F}}{\dfrac{1}{\beta r_e} + \dfrac{1}{R_F} + \dfrac{R_C}{\beta r_e R_F} + \dfrac{R_C}{R_F r_e}} = \frac{r_e\left[1 + \dfrac{R_C}{R_F}\right]}{\dfrac{1}{\beta} + \dfrac{1}{R_F}\left[r_e + \dfrac{R_C}{\beta} + R_C\right]}$$

Applying $R_C \gg r_e$ and $\dfrac{R_C}{\beta}$,

$$Z_i \cong \frac{r_e\left[1 + \dfrac{R_C}{R_F}\right]}{\dfrac{1}{\beta} + \dfrac{R_C}{R_F}} = \frac{r_e\left[\dfrac{R_F + R_C}{\cancel{R_F}}\right]}{\dfrac{R_F + \beta R_C}{\beta \cancel{R_F}}} = \frac{r_e}{\dfrac{1}{\beta}\left(\dfrac{R_F}{R_F + R_C}\right) + \dfrac{R_C}{R_C + R_F}}$$

but, since R_F typically $\gg R_C$, $R_F + R_C \cong R_F$ and $\dfrac{R_F}{R_F + R_C} = 1$

$$Z_i \cong \frac{r_e}{\dfrac{1}{\beta} + \dfrac{R_C}{R_C + R_F}} \qquad r_o \gg R_C,\ R_F > R_C$$

$$Z_o = r_o \| R_C \| R_F$$

For $r_o \geq 10R_C$,

$$Z_o \cong R_C \| R_F \qquad r_o \geq 10R_C$$

$$Z_o \cong R_C \qquad r_o \geq 10R_C, R_F \gg R_C$$

$$A_v = -\left(\frac{R_F}{R_C \| r_o + R_F}\right)\frac{R_C \| r_o}{r_e}$$

For $r_o \geq 10R_C$,

$$A_v \cong -\left(\frac{R_F}{R_C + R_F}\right)\frac{R_C}{r_e} \qquad r_o \geq 10R_C$$

and for $R_F \gg R_C$

$$A_v \cong -\frac{R_C}{r_e} \qquad r_o \geq 10R_C,\ R_F \geq R_C$$

Collector DC Feedback Configuration

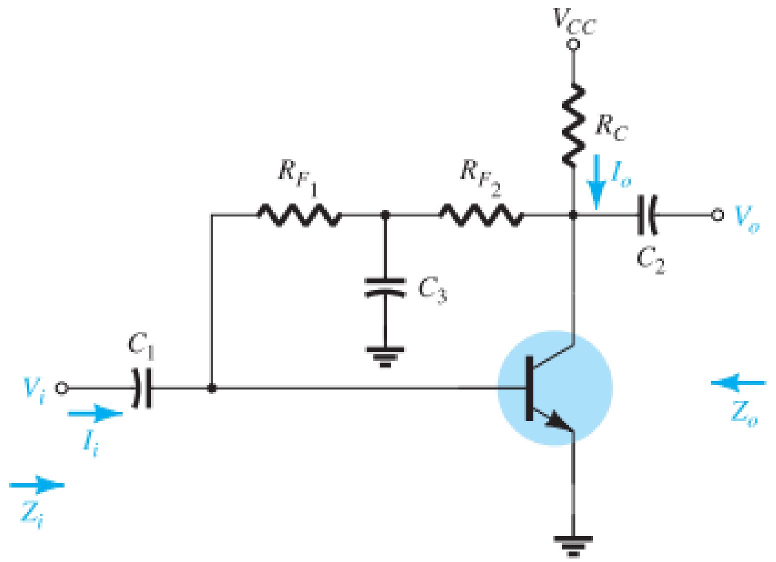


FIG. 5.50
Collector dc feedback configuration.

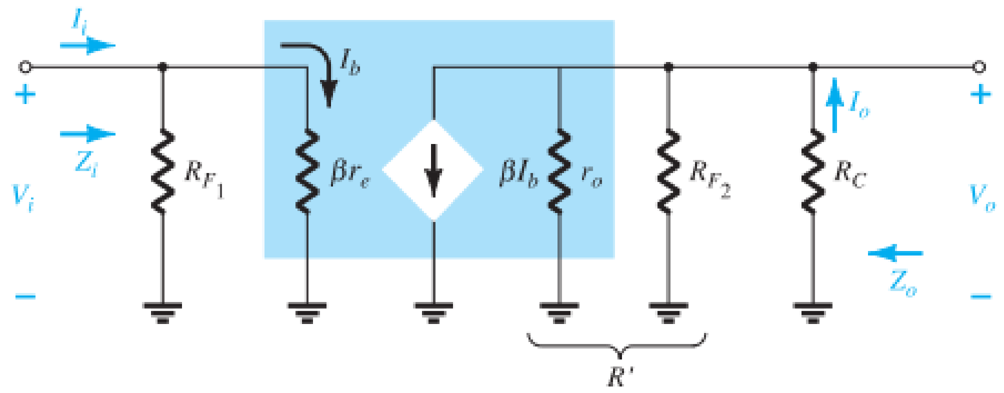


FIG. 5.51
Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.50.

$$V_o = -\beta \frac{V_i}{\beta r_e} R'$$

$$Z_i = R_{F_1} \| \beta r_e$$

$$R' = r_o \| R_{F_2} \| R_C$$

$$V_o = -\beta I_b R'$$

$$I_b = \frac{V_i}{\beta r_e}$$

$$A_v = \frac{V_o}{V_i} = -\frac{r_o \| R_{F_2} \| R_C}{r_e}$$

$$Z_o = R_C \| R_{F_2} \| r_o$$

$$Z_o \cong R_C \| R_{F_2} \qquad r_o \ge 10R_C$$

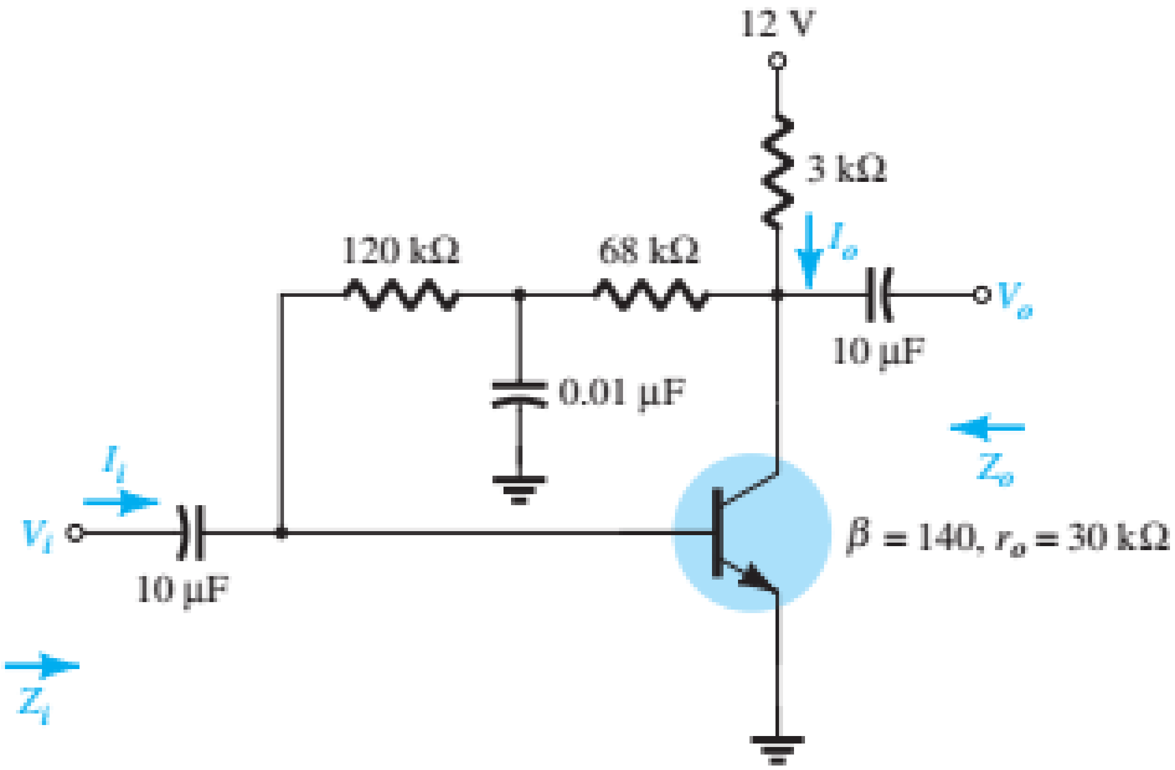
$$A_v = \frac{V_o}{V_i} \cong -\frac{R_{F_2} \| R_C}{r_e} \qquad r_o \ge 10R_C$$

180° phase shift

Collector DC Feedback Configuration (Example)

EXAMPLE 5.10 For the network of Fig. 5.52, determine:

a. r_e.
b. Z_i.
c. Z_o.
d. A_v.
e. V_o if $V_i = 2$ mV



Collector DC Feedback Configuration (Example)

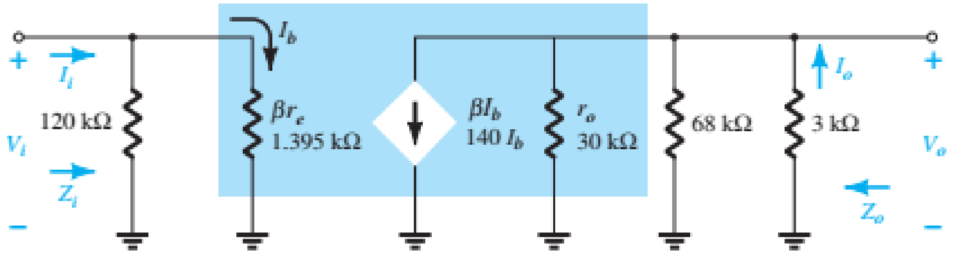


FIG. 5.53

Substituting the r_e equivalent circuit into the ac equivalent network of Fig. 5.52.

Solution:

a. DC: $I_B = \dfrac{V_{CC} - V_{BE}}{R_F + \beta R_C}$

$$= \frac{12\text{ V} - 0.7\text{ V}}{(120\text{ k}\Omega + 68\text{ k}\Omega) + (140)3\text{ k}\Omega}$$

$$= \frac{11.3\text{ V}}{608\text{ k}\Omega} = 18.6\ \mu\text{A}$$

$$I_E = (\beta + 1)I_B = (141)(18.6\ \mu\text{A})$$

$$= 2.62\text{ mA}$$

$$r_e = \frac{26\text{ mV}}{I_E} = \frac{26\text{ mV}}{2.62\text{ mA}} = \mathbf{9.92\ \Omega}$$

b. $\beta r_e = (140)(9.92\ \Omega) = 1.39\text{ k}\Omega$

The ac equivalent network appears in Fig. 5.53.

$$Z_i = R_{F_1} \| \beta r_e = 120\text{ k}\Omega \| 1.39\text{ k}\Omega$$

$$\cong \mathbf{1.37\text{ k}\Omega}$$

c. Testing the condition $r_o \geq 10R_C$, we find

$$30\text{ k}\Omega \geq 10(3\text{ k}\Omega) = 30\text{ k}\Omega$$

which is satisfied through the equals sign in the condition. Therefore,

$$Z_o \cong R_C \| R_{F_2} = 3\text{ k}\Omega \| 68\text{ k}\Omega$$

$$= \mathbf{2.87\text{ k}\Omega}$$

d. $r_o \geq 10R_C$; therefore,

$$A_v \cong -\frac{R_{F_2} \| R_C}{r_e} = -\frac{68\text{ k}\Omega \| 3\text{ k}\Omega}{9.92\ \Omega}$$

$$\cong -\frac{2.87\text{ k}\Omega}{9.92\ \Omega}$$

$$\cong \mathbf{-289.3}$$

e. $|A_v| = 289.3 = \dfrac{V_o}{V_i}$

$$V_o = 289.3V_i = 289.3(2\text{ mV}) = \mathbf{0.579\text{ V}}$$


Thank You!